Supporting Information

for

Drive-amplitude-modulation atomic force microscopy:
From vacuum to liquids

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Dynamic response in DAM-AFM
Within the point-mass approximation a cantilever can be seen as a mass on a spring. Therefore the equation of motion is given by:

$$\frac{d^2x}{dt^2} + \frac{\omega_0}{q} \frac{dx}{dt} + \omega_0^2 = A \cos(\omega_f t)$$

The general solution can be written as:

$$x(t) = \frac{A}{(\omega_0^2 - \omega_f^2) + (\frac{\omega_0}{q} \omega_f)^2} \left( \left( \frac{\omega_0^2}{\omega_f^2} \right) \left( \cos(\omega_f t) - e^{-\frac{\omega_0 t}{2q}} \cos(\omega_0 t) \right) + \frac{\omega_0}{q} \omega_f \left( \sin(\omega_0 t) - \frac{\omega_0^2 + \omega_f^2}{2\omega_0 \omega_f} e^{-\frac{\omega_0 t}{2q}} \sin(\omega_0 t) \right) \right)$$

The use of a phase-locked loop implies that $\omega_0 = \omega_f$ and thus:

$$x(t) = \frac{Aq}{\omega_0^2} \left( \sin(\omega_0 t) - e^{-\frac{\omega_0 t}{2q}} \sin(\omega_0 t) \right)$$

The standard procedure to readout the oscillation amplitude is to use a lock-in amplifier, which first multiplies the previous solution by a reference signal ($\omega_0 t$). The result of this operation is a dc component proportional to the signal amplitude plus a $2\omega_0$ component, which is usually removed with a low-pass filter. After these operations the output of the lock-in amplifier reads as:

$$A_{out} = \frac{Aq}{\omega_0^2} \left( 1 - e^{-\frac{\omega_0 t}{2q}} \right)$$

With a constant time $\tau = \frac{2q}{\omega_0}$

This corresponds to a system with a transfer function $G$:

$$G = \frac{1}{s + \frac{\omega_0}{2q}}$$

$$H = K_p$$

If feedback is performed then these values are modified as follows:
And the time response:

\[
\frac{y}{x} = \frac{G}{1 + GK}
\]

\[
\frac{y}{x} = \frac{1}{4\omega_0 \left(s + \frac{\omega_0}{2q} + \frac{K_p}{4\omega_0}\right)}
\]

As we can see, the new time constant is:

\[
\tau = \frac{4q\omega_0}{2\omega_0^2 + K_p q}
\]

This involves the gain \(K_p\) of the feedback loop. As a consequence the time response can be arbitrarily reduced by tuning the feedback gain.

This analysis was performed using a feedback that only includes a proportional gain, since this is the most important parameter to regulate the amplitude of cantilever. However the analysis could be easily generalized to include an integral or even a differential gain.