# Supporting Information

for

# A robust AFM-based method for locally measuring the elasticity of samples

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### Analytical expression for normal sample stiffness

The following computation steps establish the analytical expression for normal sample stiffness [1]. The formula is derived from the equations stated by Hurley and Turner [2] for the numerical determination of normal sample stiffness. These equations, i.e., equations (1), (2), (3), (4), (5), (6) and (7), are based on the description of the dynamics of a clamped, ideally beam-shaped cantilever elastically coupled to a sample by its tip, published by Rabe [3] and Rabe and co-workers [4]. The model takes into account characteristics of the cantilever, such as tilt angle $\alpha$ and dimensions, namely total length $L$, length from the clamped end to the tip $L_1$, length from the tip to the free end $L_2$, tip height $h$, but also characteristics of the sample, i.e., normal sample stiffness $k_{sample,norm}$ and lateral sample stiffness $k_{sample,lat}$ represented by two springs coupled to the cantilever tip, as shown in Figure S1.

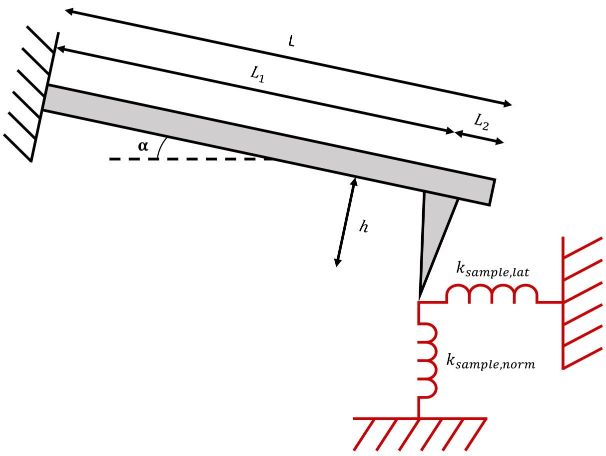


**Figure S1:** Modelization of clamped beam-shaped cantilever elastically coupled to a sample by its tip and used for the computation of normal sample stiffness. The cantilever characteristics, i.e. $L$, $L_1$, $L_2$, $h$ and $\alpha$ correspond, respectively, to cantilever length, length from the clamped end to the tip, length from the tip to the free end, tip height and tilt angle. The springs $k_{sample,norm}$ and $k_{sample,lat}$ stand, respectively, for normal and lateral sample stiffnesses.

Wavenumbers $x_n$ and $y_n$ associated with the measured $n^{\text{th}}$ flexural contact resonance $f_n$ and torsional contact resonances $t_n$ are computed by

$$x_n L = x_n^0 L \sqrt{\frac{f_n}{f_n^0}},$$

where $f_n^0$ is the cantilever resonance of $n^{th}$ flexural mode in free space and $x_n^0$ the associated wavelength (values given in Rabe [3]) obtained from the resolution of the characteristic equation $cos(x_n L)cosh(x_n L) + 1 = 0$ and

$$y_n = \frac{(2n-1)\pi}{2}\frac{t_n}{t_n^0},$$

where $t_n^0$ is the cantilever resonance of $n^{th}$ torsional mode in free space.

The normalized lateral contact stiffness is determined by

$$k_{sample,lat} = -\frac{y_n L cos(y_n L)}{sin(y_n L_1)cos(y_n L_2)} k_{lat},$$

where $k_{lat}$ is the lateral cantilever stiffness constant.

The normal contact stiffness normalized with the cantilever flexural stiffness constant $k_1$ is obtained from the expression

$$\frac{k_{sample,norm}}{k_1} = \frac{-B \pm \sqrt{B^2 - 4AC}}{6A}, \quad (1)$$

where the positive root corresponds to the normalized contact stiffness and parameters A, B and C are defined by

$$A = \frac{k_{sample,lat}}{k_{sample,norm}} A', \quad (2)$$

where

$$A' = \left(\frac{h}{L_1}\right)^2 \Big(1 - cos(x_n L_1)cosh(x_n L_1)\Big)\Big(1 + cos(x_n L_2)cosh(x_n L_2)\Big),$$

$$B = B_1 + B_2 + B_3, \tag{3}$$

with

$$B_1 = \left[ sin^2(\alpha) + \left( \frac{k_{sample,lat}}{k_{sample,norm}} \right) cos^2(\alpha) \right] B_1^{'}, \tag{4}$$

where

$$B_1^{'} = \left( \frac{h}{L_1} \right)^2 (x_n L_1)^3 \left[ \left( 1 + cos(x_n L_2) cosh(x_n L_2) \right) \left( sin(x_n L_1) cosh(x_n L_1) + cos(x_n L_1) sinh(x_n L_1) \right) \right.$$
$$\left. - \left( 1 - cos(x_n L_1) cosh(x_n L_1) \right) \left( sin(x_n L_2) cosh(x_n L_2) + cos(x_n L_2) sinh(x_n L_2) \right) \right],$$

$$B_2 = \left[ \left( \frac{k_{sample,lat}}{k_{sample,norm}} - 1 \right) cos(\alpha) sin(\alpha) \right] B_2^{'}, \tag{5}$$

where

$$B_2^{'} = 2 \left( \frac{h}{L_1} \right) (x_n L_1)^2 \left[ \left( 1 + cos(x_n L_2) cosh(x_n L_2) \right) sin(x_n L_1) sinh(x_n L_1) \right.$$
$$\left. + \left( 1 - cos(x_n L_1) cosh(x_n L_1) \right) sin(x_n L_2) sinh(x_n L_2) \right],$$

Factor 2, that does not appear in Hurley and Turner [2], is actually a necessary correction, as introduced by Steiner et al. [5],

$$B_3 = \left[cos^2(\alpha) + \left(\frac{k_{sample,lat}}{k_{sample,norm}}\right) sin^2(\alpha)\right] B_3^{'}, \tag{6}$$

where

$$B_3^{'} = x_n L_1 \Big[ \Big(1 + cos(x_n L_2) cosh(x_n L_2)\Big) \Big(sin(x_n L_1) cosh(x_n L_1) - cos(x_n L_1) sinh(x_n L_1)\Big) - \Big(1 - cos(x_n L_1) cosh(x_n L_1)\Big) \Big(sin(x_n L_2) cosh(x_n L_2) - cos(x_n L_2) sinh(x_n L_2)\Big) \Big],$$

and

$$C = 2(x_n L_1)^4 (1 + cos(x_n L) cosh(x_n L)). \tag{7}$$

Parameter B can be split into two terms, one with $k_{sample,norm}$ and the other without;

$$B = \beta_1 + \beta_2 \frac{k_{sample,lat}}{k_{sample,norm}},$$

with

$$\beta_1 = sin^2(\alpha) B_1^{'} - cos(\alpha) sin(\alpha) B_2^{'} + cos^2(\alpha) B_3^{'},$$

and

$$\beta_2 = cos^2(\alpha) B_1^{'} + cos(\alpha) sin(\alpha) B_2^{'} + sin^2(\alpha) B_3^{'}.$$

By setting a new variable $\varepsilon$ of expression

$$\varepsilon = 6A' \frac{k_{sample,lat}}{k_1},$$

we can rewrite equation (1) as

$$\varepsilon + B = \pm\sqrt{B^2 - 4AC}.$$

Finally, by squaring each side of the equation and isolating $k_{sample,norm}$, we find the following expression for the normal sample stiffness

$$k_{sample,norm} = -\frac{2A'C\varepsilon^{-1} + \beta_2}{0.5\varepsilon + \beta_1} k_{sample,lat}.$$

## References


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